

Warm-Intermediate inflationary universe model in braneworld cosmologies

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Abstract

Warm-intermediate inflationary universe models in the context of braneworld cosmologies, are studied. This study is done in the weak and strong dissipative regimes. We find that, the scalar potentials and dissipation coefficients in terms of the scalar field, evolves as type-power-law and powers of logarithms, respectively. General conditions required for these models to be realizable are derived and discussed. We also study the scalar and tensor perturbations for each regime. We use recent astronomical observations to constraint the parameters appearing in the braneworld models.

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I. INTRODUCTION

It is well known that warm inflation, as opposed to the conventional cool inflation, presents the attractive feature that it avoids the reheating period [1, 2]. The warm inflation scenario differs from the cold inflation scenario in that there is no separate reheating phase in the former, and rather radiation production occurs concurrently with inflationary expansion. In this way, warm inflation provides an alternative to the traditional reheating mechanism by smoothly connecting an early inflationary period with a radiation dominated phase. In these kind of models dissipative effects are important during the inflationary period, so that radiation production occurs concurrently together with the inflationary expansion. If the radiation field is in a highly excited state during inflation, and this has a strong damping effect on the inflaton dynamics, then, it is found a strong regimen of warm inflation. Also, the dissipating effect arises from a friction term which describes the processes of the scalar field dissipating into a thermal bath via its interaction with other fields. Warm inflation shows how thermal fluctuations during inflation may play a dominant role in producing the initial fluctuations necessary for Large-Scale Structure (LSS) formation. In this way, density fluctuations arise from thermal rather than quantum fluctuations [3, 4]. Among the most attractive features of these models, warm inflation ends at the epoch when the universe stops inflating and "smoothly" enters in a radiation dominated Big-Bang phase[1]. The matter components of the universe are created by the decay of either the remaining inflationary field or the dominant radiation field [5].

On the other hand, a possible evolution during inflation is the particular scenario of intermediate inflation, in which the scale factor, $a(t)$, evolves as $a = \exp(At^f)$, where A and f are two constants, where $0 < f < 1$; the expansion of this universe is slower than standard de Sitter inflation ($a = \exp(Ht)$), but faster than power law inflation ($a = t^p; p > 1$)[6], this is the reason why it is called "intermediate". This model was introduced as an exact solution for a particular scalar field potential of the type $V(\phi) \propto \phi^{-4(f^{-1}-1)}$ [7]. In the slow-roll approximation, and with this sort of potential, it is possible to have a spectrum of density perturbations which presents a scale-invariant spectral index, i.e. $n_s = 1$, the so-called Harrison-Zel'dovich spectrum provided that f takes the value of $2/3$ [8]. Even though this kind of spectrum is disfavored by the current WMAP data[9, 10], the inclusion of tensor perturbations, which could be present at some point by inflation and parametrized by the

tensor-to-scalar ratio r , the conclusion that $n_s \geq 1$ is allowed providing that the value of r is significantly nonzero[11]. In fact, in Ref. [12] was shown that the combination $n_s = 1$ and $r > 0$ is given by a version of the intermediate inflation in which the scale factor varies as $a(t) \propto e^{t^{2/3}}$ within the slow-roll approximation. Eventually, warm inflationary universe models in the context of intermediate inflation in General Relativity was studied in Ref.[13] and inflation intermediate on the brane was considered in Ref.[14], where the value of the tensor-scalar ratio r is significantly nonzero.

In view of these observations, it is important to further our understanding of the inflationary models from a theoretical perspective. Therefore, there is considerable interest in inflationary scenarios motivated by superstring and M-theory (see Refs.[15, 16]). Specifically, the braneworld scenario, where our observable, fourdimensional universe is regarded as a domain wall embedded in a higherdimensional bulk space [17]. This theory suggests that in order to have a ghost-free action high order curvature invariant corrections to the Einstein-Hilbert action must be proportional to the Gauss-Bonnet (GB) term[18]. The GB terms arise naturally as the leading order of the α expansion to the low-energy string effective action, where α is the inverse string tension[19]. This kind of theory has been applied to possible resolution of the initial singularity problem[20], to the study of Black-Hole solutions[21], accelerated cosmological solutions[22]. Recently, accelerated expansion in an intermediate inflationary universe models using the GB brane was studied in Ref.[23] and in a closed or open inflationary universe models using the GB brane was considered in Ref.[24]. In particular, in Ref.[25], it has been found that for a dark energy model the GB interaction in four dimensions with a dynamical dilatonic scalar field coupling leads to a solution of the form $a \propto \exp At^f$, where the universe starts evolving with a decelerated exponential expansion.

In this way, the idea that inflation, or specifically, intermediate inflation, comes from an effective theory at low dimension of a more fundamental string theory is in itself very appealing. Thus, our aim in this paper is to study an evolving intermediate scale factor in the warm inflationary universe scenario in the context of braneworld cosmologies. We will do this for two regimes; the weak and the strong dissipative regimes.

The outline of the paper is as follows. The next section presents a short review of the modified Friedmann equation and the warm-intermediate inflationary phase in braneworld cosmologies. In the Sections III and IV we discuss the weak and strong dissipative regimes,

respectively. Here, we give explicit expressions for the dissipative coefficient, the scalar power spectrum and the tensor-scalar ratio. Finally, our conclusions are presented in Section V. We chose units so that $c = \hbar = 1$.

II. THE WARM-INTERMEDIATE INFLATIONARY PHASE IN GAUSS BONNET.

We start with the five-dimensional bulk action for the GB braneworld:

$$S = \frac{1}{2\kappa_5^2} \int_{bulk} d^5x \sqrt{-g_5} \{ R - 2\Lambda_5 + \alpha (R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} - 4R^{\mu\nu} R_{\nu\mu} + R^2) \} \\ + \int_{brane} d^4x \sqrt{-g_4} (\mathcal{L}_{matter} - \sigma), \quad (1)$$

where $\Lambda_5 = -3\mu^2(2 - 4\alpha\mu^2)$ is the cosmological constant in five dimensions, with the AdS_5 energy scale μ , α is the GB coupling constant, $\kappa_5 = 8\pi/m_5$ is the five dimensional gravitational coupling constant and σ is the brane tension. \mathcal{L}_{matter} is the matter lagrangian for the inflaton field on the brane. We will consider the case that a perfect fluid matter source with density ρ is confined to the brane.

A Friedmann-Robertson-Walker (FRW) brane in an AdS_5 bulk is a solution to the field and junction equations (see Refs.[26–28]). The modified Friedmann on the brane can be written as

$$H^2 = \frac{1}{4\alpha} \left[(1 - 4\alpha\mu^2) \cosh \left(\frac{2\chi}{3} \right) - 1 \right], \quad (2)$$

$$\kappa_5^2(\rho + \sigma) = \left[\frac{2(1 - 4\alpha\mu^2)^3}{\alpha} \right]^{1/2} \sinh \chi, \quad (3)$$

where χ represents a dimensionless measure of the energy density ρ . In this work we will assume that the matter fields are restricted to a lower dimensional hypersurface (brane) and that gravity exists throughout the space-time (brane and bulk) as a dynamical theory of geometry. Also, for 4D homogeneous and isotropic Friedmann cosmology, an extended version of Birkhoffs theorem tells us that if the bulk space-time is AdS , it implies that the effect of the Weyl tensor (known as dark radiation) does not appear in the modified Friedmann equation. On the other hand, the brane Friedmann equation for the general, where the bulk space-time may be interpreted as a charged black hole was studied in Refs.[29–31].

The modified Friedmann equation (2), together with Eq. (3), shows that there is a characteristic GB energy scale[32], where $m_{GB} = \left[\frac{2(1-4\alpha\mu^2)^3}{\alpha\kappa_5^4} \right]^{1/8}$, such that the GB high energy regime ($\chi \gg 1$) occurs if $\rho + \sigma \gg m_{GB}^4$. Expanding Eq. (2) in χ and using Eq.(3), we find in the full theory three regimes for the dynamical history of the brane universe [26–28]:

$$\rho \gg m_{GB}^4 \Rightarrow H^2 \approx \left[\frac{\kappa_5^2}{16\alpha} \rho \right]^{2/3} \quad (GB), \quad (4)$$

$$m_{GB} \gg \rho \gg \sigma \Rightarrow H^2 \approx \frac{\kappa_4^2}{6\sigma} \rho^2 \quad (RS), \quad (5)$$

$$\rho \ll \sigma \Rightarrow H^2 \approx \frac{\kappa_4^2}{3} \rho \quad (GR). \quad (6)$$

Clearly Eqs. (4), (5) and (6) are much simpler than the full Eq (2) and in a practical case one of the three energy regimes will be assumed. Therefore, can be useful to describe the universe in a region of time and energy in which [33, 34]

$$H^2 = \beta_q^2 \rho^q, \quad (7)$$

where $H = \dot{a}/a$ is the Hubble parameter and q is a patch parameter that describes a particular cosmological model under consideration. The choice $q = 1$ corresponds to the standard General Relativity (GR) with $\beta_1^2 = 8\pi/3m_p^2 = \kappa^2/3$, where m_p is the four dimensional Planck mass. If we take $q = 2$, we obtain the high energy limit of the brane world cosmology, Randall-Sundrum (RS), in which $\beta_2^2 = 4\pi/3\sigma m_p^2 = \kappa^2/6\sigma$. Finally, for $q = 2/3$, we have the GB brane world cosmology, with $\beta_{2/3}^2 = G_5/16\zeta$, where G_5 is the 5D gravitational coupling constant and $\zeta = 1/8g_s$ is the GB coupling (g_s is the string energy scale). The parameter q , which describes the effective degrees of freedom from gravity, can take a value in a non-standard set because of the introduction of non-perturbative stringy effects. Here, we mentioned some possibilities, for instance, in Ref.[35] it was found that an appropriate region to a patch parameter q is given by $1/2 = q < \infty$. On the other hand, from Cardassian cosmology it is possible to obtain a Friedmann equation similar (7) as a consequence of embedding our observable universe as a 3+1 dimensional brane in extra dimensions. In particular, a modified FRW equation was obtained in our observable brane with $H^2 \propto \rho^n$ for any n in Ref.[36].

On the other hand, we neglect any contribution from both the Weyl tensor and the brane-bulk exchange, assuming there is some confinement mechanism for a perfect fluid. Thus, the

energy conservation equation on the brane follows directly from the Gauss-Codazzi equations. For a perfect fluid matter source it is reduced to the familiar form, $\dot{\rho} + 3H(\rho + P) = 0$, where P represent the pressure density. The dot denotes derivative with respect to the cosmological time t .

In the following we will consider a total energy density $\rho = \rho_\phi + \rho_\gamma$, where ϕ corresponds to a self-interacting scalar field with energy density, ρ_ϕ , given by $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, $V(\phi) = V$ is the scalar potential and ρ_γ represents the radiation energy density.

From Eq.(7), we assume that the gravitational dynamics give rise to a Friedmann equation of the form

$$H^2 = \beta_q^2 [\rho_\phi + \rho_\gamma]^q. \quad (8)$$

The dynamics of the cosmological model, for ρ_ϕ and ρ_γ in the warm inflationary scenario is described by the equations

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Gamma \dot{\phi}^2, \quad (9)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma \dot{\phi}^2. \quad (10)$$

Here Γ is the dissipation coefficient and it is responsible of the decay of the scalar field into radiation during the inflationary era. Γ can be assumed to be a constant or a function of the scalar field ϕ , or the temperature T , or both [1]. On the other hand, Γ must satisfy $\Gamma > 0$ by the Second Law of Thermodynamics.

During the inflationary epoch the energy density associated to the scalar field dominates over the energy density associated to the radiation field[1, 3], i.e., $\rho_\phi > \rho_\gamma$ and the Friedmann equation (8) reduces to

$$H^2 \approx \beta_q^2 \rho_\phi^q, \quad (11)$$

and from Eqs. (9) and (11), we can write

$$\dot{\phi}^2 = -\frac{2H^{\frac{2}{q}-2}\dot{H}}{3q\beta_q^{\frac{2}{q}}(1+R)}, \quad (12)$$

where R is the rate defined as

$$R = \frac{\Gamma}{3H}. \quad (13)$$

For the weak (strong) dissipation regime, we have $R < 1$ ($R > 1$) (see Refs.[1, 3]).

We also consider that during warm inflation the radiation production is quasi-stable[1, 3], i.e. $\dot{\rho}_\gamma \ll 4H\rho_\gamma$, $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$ and from Eq.(10) we obtained that

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H} = -\frac{\Gamma H^{\frac{2}{q}-3}\dot{H}}{6q\beta_q^{\frac{2}{q}}(1+R)}, \quad (14)$$

which could be written as $\rho_\gamma = C_\gamma T^4$, where $C_\gamma = \pi^2 g_*/30$ and g_* is the number of relativistic degrees of freedom. Here T is the temperature of the thermal bath.

From Eqs.(12) and (14) we get that

$$T = \left[-\frac{\Gamma H^{\frac{2}{q}-3}\dot{H}}{6 C_\gamma q \beta_q^{\frac{2}{q}}(1+R)} \right]^{1/4}. \quad (15)$$

On the other hand, in warm inflation the interactions are important during the inflationary scenario. If the fields interacting with the inflaton are at high temperature, then it is complex to control the thermal loop corrections to the effective potential that is required to preserve the appropriate flat potential required for inflation. Nevertheless, if the fields interacting with the inflaton are at low temperature, then supersymmetry can be applied to cancel the quantum radiative corrections, and maintain an appropriate potential [37].

From first principles in quantum field theory the dissipation coefficient Γ is computed for models in cases of low-temperature regimes[37] (see also Ref.[38]). Here, was developed the dissipation coefficients in supersymmetric models which have an inflaton together with multiplets of heavy and light fields. In this approach, it was used an interacting supersymmetric theory, which has three superfields Φ , X and Y with a superpotential, $W = g\Phi X^2 + hXY^2$, where ϕ , χ and y refer to their bosonic component. The interaction structure for this superpotential is habitual in many particle physics SUSY models during inflation. Also, this superpotential can simply be modified to develop a hybrid inflationary model. The inflaton field couples to heavy bosonic field χ and fermions ψ_χ , obtain their masses through couplings to ϕ , where $m_{\psi_\chi} = m_\chi = g\phi$. In the low -temperature regime, i.e. $m_\chi, m_{\psi_\chi} > T > H$, the dissipation coefficient, when X and Y are singlets, becomes [37]

$$\Gamma \simeq 0.64 g^2 h^4 \left(\frac{g\phi}{m_\chi} \right)^4 \frac{T^3}{m_\chi^2}. \quad (16)$$

This latter equation can be rewritten as

$$\Gamma \simeq C_\phi \frac{T^3}{\phi^2}, \quad (17)$$

where $C_\phi = 0.64 h^4 \mathcal{N}$. Here $\mathcal{N} = \mathcal{N}_\chi \mathcal{N}_{decay}^2$, where \mathcal{N}_χ is the multiplicity of the X superfield and \mathcal{N}_{decay} is the number of decay channels available in X 's decay [37, 39].

From Eq.(15) the above equation becomes

$$\Gamma^{1/4} (1 + R)^{3/4} \simeq \left[-\frac{H^{\frac{2}{q}-3} \dot{H}}{6 C_\gamma q \beta_q^{\frac{2}{q}}} \right]^{3/4} \frac{C_\phi}{\phi^2}, \quad (18)$$

which determines the dissipation coefficient in the weak (or strong) dissipative regime in terms of scalar field ϕ and the parameters of the model.

We should note that in general the scalar potential from Eqs.(8) and (14) becomes

$$V(\phi) = \left[\frac{H}{\beta_q} \right]^{\frac{2}{q}} \left[1 + \frac{\dot{H}}{3 q H^2 (1 + R)} \left(1 + \frac{3}{2} R \right) \right], \quad (19)$$

which could be expressed explicitly in terms of the scalar field, ϕ , by using Eqs.(12) and (18), in the weak (or strong) dissipative regime.

On the other hand, solutions can be found for warm-intermediate inflationary universe models where the scale factor, $a(t)$, expands as follows

$$a(t) = \exp(A t^f). \quad (20)$$

In the following, we develop models for a variable dissipation coefficient Γ , and we will restrict ourselves to the weak (or strong) dissipation regime.

III. THE WEAK DISSIPATIVE REGIME.

Assuming that, once the system evolves according to the weak dissipative regime, i.e. $\Gamma < 3H$, it remains in such limit for the rest of the evolution. From Eqs.(12) and (20), we obtained a relation between the scalar field and cosmological times given by

$$\phi(t) = \phi_0 + \frac{C}{2\theta} t^\theta, \quad (21)$$

where $C = \left[\frac{8(Af)^{\frac{2}{q}-1}(1-f)}{3q\beta_q^{\frac{2}{q}}} \right]^{1/2}$, and $\theta = \frac{1}{2} \left[(f-1)(\frac{2}{q}-1) + 1 \right]$.

Here $\phi(t=0) = \phi_0$. The Hubble parameter as a function of the inflaton field, ϕ , results in

$$H(\phi) = A f \left[\frac{2\theta}{C} \right]^{(f-1)/\theta} (\phi - \phi_0)^{(f-1)/\theta}. \quad (22)$$

Without loss of generality ϕ_0 can be taken to be zero.

From Eq.(18) we obtain for the dissipation coefficient as function of scalar field

$$\Gamma(\phi) = D \phi^{-\alpha_1}, \quad (23)$$

where $D = (Af)^{6(\frac{1}{q}-1)} C_\phi^4 \left\{ \left[\frac{(1-f)}{6 C_\gamma q \beta_q^{\frac{q}{2}}} \right] \left[\left(\frac{2\theta}{C} \right)^{\frac{2(\frac{1}{q}-1)(f-1)-1}{\theta}} \right] \right\}^3$, and $\alpha_1 = 2 + \frac{3}{\theta}(f+1)$.

Using the slow-roll approximation, $\dot{\phi}^2/2 < V(\phi)$, and $V(\phi) > \rho_\gamma$, the scalar potential given by Eq.(19) reduces to

$$V(\phi) \simeq \left(\frac{H}{\beta_q} \right)^{\frac{2}{q}} = V_0 \phi^{-\alpha_2}, \quad (24)$$

where the constants V_0 and α_2 are; $V_0 = \left[\frac{Af(\frac{2\theta}{C})^{\frac{f-1}{\theta}}}{\beta_q} \right]^{2/q}$ and $\alpha_2 = \frac{2(1-f)}{q\theta}$, respectively. Note that this kind of potential does not present a minimum. Note also that the scalar field ϕ , the Hubble parameter H , and the scalar potential $V(\phi)$ become independent of the parameters C_ϕ and C_γ .

Introducing the Hubble slow-roll parameters (ϵ_1, η_η) and potential slow-roll parameters (ϵ_1^q, η_n^q) , see Ref.[40], we write

$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \epsilon_1^q = \frac{qV'^2}{6\beta_q^2 V^{q+1}} = \frac{(1-f)}{Af} \left(\frac{C}{2\theta} \right)^{f/\theta} \frac{1}{\phi^{f/\theta}}, \quad (25)$$

$$\eta_n = -\frac{1}{H^n \dot{\phi}} \frac{d^{n+1}\phi}{dt^{n+1}} = \eta_n^q, \quad (26)$$

and in particular

$$\eta_1^q = \frac{1}{3\beta_q^2} \left[\frac{V''}{V^q} - \frac{qV'^2}{2V^{q+1}} \right] = \frac{(2-f)}{Af} \left(\frac{C}{2\theta} \right)^{f/\theta} \frac{1}{\phi^{f/\theta}},$$

and

$$\eta_2^q = \frac{-1}{(3\beta_q^2)^2} \left[\frac{V'V''}{V^{2q}} + \frac{(V'')^2}{V^{2q}} - \frac{5qV''(V')^2}{V^{2q+1}} + \frac{q(q+2)(V')^4}{2V^{2(q+1)}} \right].$$

So, the condition for inflation to occur $\ddot{a} > 0$ (or equivalently $\epsilon_1^q < 1$) is only satisfied when $\phi^{f/\theta} > \frac{(1-f)}{Af} \left(\frac{C}{2\theta} \right)^{f/\theta}$.

The number of e-folds between two different values of cosmological times t_1 and t_2 (or equivalently between two values ϕ_1 and ϕ_2 of the scalar field) is given by

$$N = \int_{t_1}^{t_2} H dt = A(t_2^f - t_1^f) = A \left(\frac{2\theta}{C} \right)^{f/\theta} (\phi_2^{f/\theta} - \phi_1^{f/\theta}). \quad (27)$$

Here we have used Eq.(21).

If we assume that inflation begins at the earliest possible stage (see Ref.[12]), that is, at $\epsilon_1^q = 1$ (or equivalently $\ddot{a} = 0$), the scalar field becomes

$$\phi_1 = \left(\frac{1-f}{Af} \right)^{\theta/f} \left(\frac{C}{2\theta} \right). \quad (28)$$

On the other hand, as argued in Refs.[1, 41], the amplitude of scalar perturbations generated during inflation for a flat space is approximately $\mathcal{P}_{\mathcal{R}}^{1/2} = \frac{H}{\dot{\phi}} \delta\phi$. In particular in the warm inflation regime, a thermalized radiation component is present, therefore, inflation fluctuations are dominantly thermal rather than quantum. In the weak dissipation limit, we have $\delta\phi^2 \simeq H T$ [3, 42]. From Eqs.(12) and (15), $\mathcal{P}_{\mathcal{R}}$ becomes

$$\mathcal{P}_{\mathcal{R}} \simeq \left[\frac{3^3 q^3 \beta_q^{\frac{6}{q}} \Gamma}{2^5 C_\gamma} \right]^{1/4} \left[\frac{H^{\frac{17}{3}-\frac{2}{q}}}{-\dot{H}} \right]^{3/4} = Q \phi^{\alpha_4}, \quad (29)$$

where $Q = \left[\left(\frac{Af}{2} \right)^8 \left(\frac{C_\phi}{C_\gamma} \right)^4 \left(\frac{2\theta}{C} \right)^{\frac{8(f-1)}{\theta}} \right]^{1/4}$ and $\alpha_4 = \frac{1}{\theta} \left[(3f-4) + \frac{2(1-f)}{q} \right]$.

The scalar spectral index n_s is given by $n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$, where the interval in wave number is related to the number of e-folds by the relation $d \ln k(\phi) = dN(\phi) = (H/\dot{\phi}) d\phi$. From Eqs. (21) and (29), we get,

$$n_s = 1 + I \phi^{\alpha_5}, \quad (30)$$

where $I = \left(\frac{2}{3 q \beta_q^{\frac{q}{q}} \frac{2}{q}} \right)^{1/2} \alpha_4 (Af)^{\frac{1}{q}-\frac{3}{2}} (1-f)^{\frac{1}{2}} \left(\frac{2\theta}{C} \right)^{\frac{1}{\theta} \left[\frac{f-1}{q} + 1 - \frac{3f}{2} \right]}$ and $\alpha_5 = \frac{1}{\theta} \left[\frac{f-1}{q} + 1 - \left(\theta + \frac{3f}{2} \right) \right]$.

Note that the scalar spectral index can be re-expressed in terms of the number of e-folding, N . By using Eqs.(27) and (28) we have

$$n_s = 1 + \frac{3f-4 + \frac{2(1-f)}{q}}{[1+f(N-1)]}, \quad (31)$$

and the value of f in terms of the n_s and N becomes

$$f = \frac{\frac{2}{q} - (n_s + 3)}{N(n_s - 1) - (n_s + 2) + \frac{2}{q}}.$$

In particular, for GB brane world cosmology ($q = 2/3$), $n_s = 0.96$ and $N = 60$ we obtain that $f \simeq 0.12$.

From Eqs.(27), (28), (29) and (30), we can write the parameter A in terms of the particle physics parameters C_γ and C_ϕ , in the form

$$A = \left(\frac{4\{1 + f(N-1)\}^{\frac{\alpha_4}{\alpha_5}} C_\gamma \mathcal{P}_\mathcal{R}}{f^{(2+\frac{\alpha_4}{\alpha_5})} C_\phi} \right)^{\alpha_7} M, \quad (32)$$

where $M = \left[\frac{3 q \beta_q^{\frac{2}{q}} \theta^2}{2 f^{\frac{2}{q}-1} (1-f)} \right]^{\alpha_7 \left[\frac{\alpha_4}{2\alpha_5} \left\{ \frac{1}{\theta} \left(\frac{f-1}{q} + 1 - \frac{3f}{2} \right) - 1 \right\} - \frac{f-1}{\theta} \right]}$, $\alpha_7 = \frac{f}{\alpha_6 \left\{ (3f-4) + \frac{2(1-f)}{q} \right\} - \alpha_5}$, and $\alpha_6 = \frac{1}{q} - \frac{1}{2} \left\{ 3 + \frac{1}{\theta} \left(\frac{2}{q} - 1 \right) \left(\frac{f-1}{q} + 1 - \frac{3f}{2} \right) \right\}$.

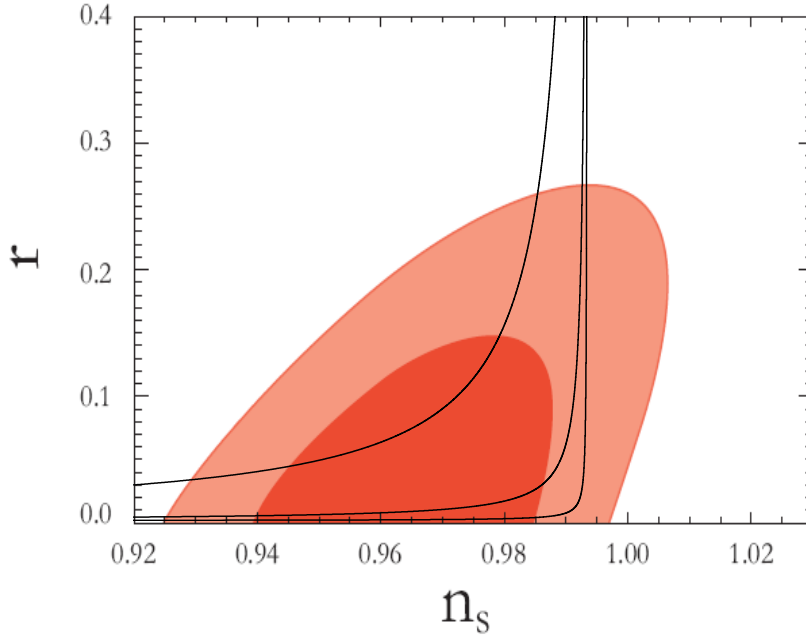


FIG. 1: Evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s in the weak dissipative regime, for the GB cosmology ($q = 2/3$) and three different values of the parameter C_ϕ . Here we used $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$, and $\beta_{2/3}^2 = 10^{-3} m_p^{-2/3}$.

In the following we will consider that the general expression for the amplitude of scalar perturbations in GB brane world is given by[32]

$$\mathcal{P}_{\mathcal{R},GB} = \mathcal{P}_\mathcal{R} G_\beta^2(H/\mu) = Q \phi^{\alpha_4} G_\beta^2(H/\mu), \quad (33)$$

where the GB brane world correction is given by

$$G_\beta^2(x) = \left[\frac{3(1+\beta)x^2}{2(3-\beta+2\beta x^2)\sqrt{1+x^2}+2(\beta-3)} \right]^3,$$

where $x \equiv H\mu$ is a dimensionless measure of energy scale, and $\beta = 4\alpha\mu$. The RS amplification factor is recovered when $\beta = 0$ [43].

As it was mentioned in Ref.[44] the generation of tensor perturbations during inflation would produce gravitational wave. In order to confront these models with observations, we need to consider the q -tensor-scalar ratio $r_q = 16 A_{T,q}^2/A_{S,q}^2$, where the q -scalar amplitude is normalized by $A_{S,q}^2 = 4\mathcal{P}_\mathcal{R}/25$. Here, the tensor amplitude is given by

$$A_{T,q}^2 = A_{T,GR}^2 F_\beta^2(H/\mu), \quad (34)$$

where $A_{T,GR}^2$ is the standard amplitude in GR i.e., $A_{T,GR} = \sqrt{24}\beta_1 (H/2\pi)$, and the function F_β contains the information about the GB term [32]

$$F_\beta^{-2} = \sqrt{1+x^2} - \left(\frac{1-\beta}{1+\beta} \right) x^2 \sinh^{-1} \left(\frac{1}{x} \right) \quad (x \equiv \frac{H}{\mu}).$$

Following, Ref.[33] we approximate the function $F_\beta^2 \approx F_q^2$, where for the GR regime $F_{q=1}^2 \approx F_\beta^2(H/\mu \ll 1) = 1$, for the RS regime $F_{q=2}^2 \approx F_{\beta=0}^2(H/\mu \gg 1) = 3H/(2\mu)$, and finally for the GB regime $F_{q=2/3}^2 \approx F_\beta^2(H/\mu \gg 1) = (1+\beta)H/(2\beta\mu)$. The tensor amplitude up to leading-order is given by

$$A_{T,q}^2 = \frac{3q\beta_q^{2-2(1-q^{-1})}}{(5\pi)^2} \frac{H^{2+2(1-q^{-1})}}{2\zeta_q}, \quad (35)$$

with $\zeta_{q=1} = \zeta_{q=2/3} = 1$ and $\zeta_{q=2} = \frac{2}{3}$ [40]. Finally, the q -tensor-scalar ratio from Eqs.(33) and (35) becomes

$$r_q = 16 \frac{A_{T,q}^2}{A_{S,q}^2} = \frac{16}{\zeta_q} \frac{(2-f)}{A f} \left(\frac{C}{2\theta} \right)^{f/\theta} \frac{1}{\phi^{f/\theta} G_\beta^2(\phi)}, \quad (36)$$

in our cosmological models.

In particular, the Fig.1 we show the dependence of the tensor-scalar ratio r on the spectral index n_s for the GB regime, where $q = 2/3$. From left to right $C_\phi = 10^8$, $C_\phi = 10^9$ and $C_\phi = 10^{10}$, respectively. From Ref.[45], two-dimensional marginalized constraints (68% and 95% confidence levels) on inflationary parameters r and n_s , the spectral index of fluctuations, defined at $k_0 = 0.002 \text{ Mpc}^{-1}$. The seven-year WMAP data places stronger limits on r . In

order to write down values that relate n_s and r , we used Eqs. (30) and (36). Also we have used the values $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$, and $\beta_{2/3}^2 = 10^{-5} m_p^{-2/3}$, respectively. From Eqs.(27) and (36), we observed numerically that for the GB regime and $C_\phi = 10^8$, the curve $r = r(n_s)$ (see Fig.1) for WMAP-7 years enters the 95% confidence region for $r \simeq 0.26$, which corresponds to the number of e-folds, $N \simeq 228$. The curve $r = r(n_s)$ enters the 68% confidence region for $r \simeq 0.15$ corresponds to $N \simeq 128$, in this way the GB regime is viable for large values of the number of e-folds N . We also noted that the parameter C_ϕ , which is bounded from bellow, $C_\phi > 10^7$, the GB regime is well supported by the data as could be seen from Fig.(1).

IV. THE STRONG DISSIPATIVE REGIME.

We consider now the case in which Γ is large enough for the system to remain in strong dissipation until the end of inflation, i.e. $R > 1$. From Eqs.(12) and (20), we can obtained a relation between the scalar field and cosmological times given by

$$\phi(t) = \phi_0 \exp[\tau_1 t^{\mu_1}], \quad (37)$$

where $\phi(t=0) = \phi_0$, τ_1 and μ_1 are defined by

$$\tau_1 = \frac{1}{\mu_1} \left[\frac{2^7 C_\gamma^3 (1-f)}{q \beta_q^{\frac{2}{q}}} \right]^{1/8} \frac{(A f)^{\frac{1}{4}(\frac{1}{q} + \frac{3}{2})}}{C_\phi^{1/2}} \quad \text{and} \quad \mu_1 = \frac{1}{4} \left[\frac{f-2}{2} + (f-1)\left(\frac{1}{q} + 1\right) \right] + 1.$$

The Hubble parameter as a function of the inflaton field, ϕ , result as

$$H(\phi) = A f \left[\frac{1}{\tau_1} \ln(\phi/\phi_0) \right]^{-(1-f)/(\mu_1)}. \quad (38)$$

Without loss of generality we can taken $\phi_0 = 1$.

From Eq.(18) the dissipation coefficient, Γ , can be expressed as a function of the scalar field, ϕ , as follows

$$\Gamma(\phi) = \frac{\tau_2}{\phi^2} [\ln(\phi)]^{-\mu_2}, \quad (39)$$

where $\tau_2 = \left[\frac{(1-f)(A f)^{\frac{2}{q}-1}}{2 C_\gamma q \beta_q^{\frac{2}{q}}} \right]^{3/4} C_\phi \tau_1^{\mu_2}$ and $\mu_2 = \frac{3}{4\mu_1} \left[\frac{2(1-f)}{q} + f \right]$.

From Eq.(19) the scalar potential $V(\phi)$, becomes

$$V(\phi) \simeq \left[\frac{A f}{\beta_q} \right]^{2/q} \left[\frac{1}{\tau_1} \ln(\phi) \right]^{-\frac{2}{q}(\frac{1-f}{\mu_1})}, \quad (40)$$

and as in the previous case, this kind of potential does not present a minimum.

In this regime the dimensionless slow-roll parameters are

$$\epsilon_1^q = -\frac{\dot{H}}{H^2} = \left(\frac{1-f}{A f} \right) \left[\frac{\tau_1}{\ln(\phi)} \right]^{\frac{f}{\mu_1}}, \quad (41)$$

and

$$\eta_1^q = \left(\frac{2-f}{A f} \right) \left[\frac{\tau_1}{\ln(\phi)} \right]^{\frac{f}{\mu_1}}. \quad (42)$$

Imposing the condition $\epsilon_1^q = 1$ at the beginning of inflation (see Ref.[12]), the scalar field ϕ , takes at this time the value

$$\phi_1 = \exp \left(\tau_1 \left[\frac{1-f}{A f} \right]^{\frac{\mu_1}{f}} \right). \quad (43)$$

The number of e-folds becomes given by

$$N = \int_{t_1}^{t_2} H dt = A \tau_1^{-\frac{f}{\mu_1}} \left[(\ln \phi_2)^{\frac{f}{\mu_1}} - (\ln \phi_1)^{\frac{f}{\mu_1}} \right], \quad (44)$$

where Eq.(37) was used.

In this regime and following Ref.[46], we can write $\delta\phi^2 \simeq \frac{k_F T}{2\pi^2}$, where the wave-number k_F is defined by $k_F = \sqrt{\Gamma H} = H \sqrt{3R} > H$, and corresponds to the freeze-out scale at which dissipation damps out to the thermally excited fluctuations. From Eqs.(37) and (39) we obtained that

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{1}{2\pi^2} \left[\frac{\Gamma^3 H^9}{4 C_\gamma \dot{\phi}^6} \right]^{1/4} \simeq \frac{1}{4\pi^2} \left[\frac{\Gamma^6 q^3 \beta_q^{\frac{6}{q}} H^{12-\frac{6}{q}}}{2 C_\gamma (-\dot{H})^3} \right]^{1/4} \simeq v_1 \frac{(\ln \phi)^{v_3}}{\phi^3}, \quad (45)$$

where $v_1 = \frac{1}{4\pi^2} \left[\frac{q^3 \beta_q^{\frac{6}{q}} \tau_2^6 (A f)^{9-\frac{6}{q}} \tau_1^{v_2}}{2 C_\gamma (1-f)^3} \right]^{1/4}$, $v_2 = 24 \frac{[2(f+q-1)-3fq]}{[2(2q-1)+f(2+3q)]}$, and $v_3 = 3 \frac{[f(2+3q)-2(1+2q)]}{[f(2+3q)-2(1-2q)]}$.

From Eq.(45) the scalar spectral index $n_s = d\mathcal{P}_{\mathcal{R}}/d \ln k$, is given by

$$n_s \simeq 1 - \left[\frac{3 \ln(\phi) - v_3}{\gamma_6 \ln(\phi)^{\gamma_5}} \right], \quad (46)$$

where $\gamma_5 = \frac{8fq}{2(2q-1)+f(2+3q)}$, $\gamma_6 = \frac{(A f)^{\frac{5}{8}-\frac{1}{4q}}}{(1-f)^{1/8}} \left[C_\phi q \beta_q^{\frac{2}{q}} \left(\frac{1}{2 C_\gamma q \beta_q^{\frac{q}{q}}} \right)^{3/4} \right]^{1/2} \tau_1^{\gamma_7}$ and $\gamma_7 = \frac{f(2-5q)+4q-2}{4q-2+f(2+3q)}$.

The scalar spectra index n_s also can be write in terms of the number of e-folds N . Thus, using Eqs. (43) and (44), we get

$$n_s \simeq 1 - f A \left[\frac{3 \tau_1 [1 + f(N-1)]^{\frac{1}{\gamma_5}} (f A)^{-\frac{1}{\gamma_5}} - v_3}{\gamma_6 [1 + f(N-1)] \tau_1^{\gamma_5}} \right]. \quad (47)$$

For the strong dissipative regime we may write the q-tensor-scalar ratio as

$$r_q = 24 r_0 \left[\frac{\phi^3}{(\ln \phi)^{v_3+2(1-f)/\mu_1}} \right] \frac{F_\beta^2(x)}{G_\beta^2(x)}, \quad (48)$$

where $r_0 = \left[\frac{25 \beta_1^2 A^2 f^2 \tau_1^{2(1-f)/\mu_1}}{v_1 \pi^2} \right]$ and $x = H/\mu$.

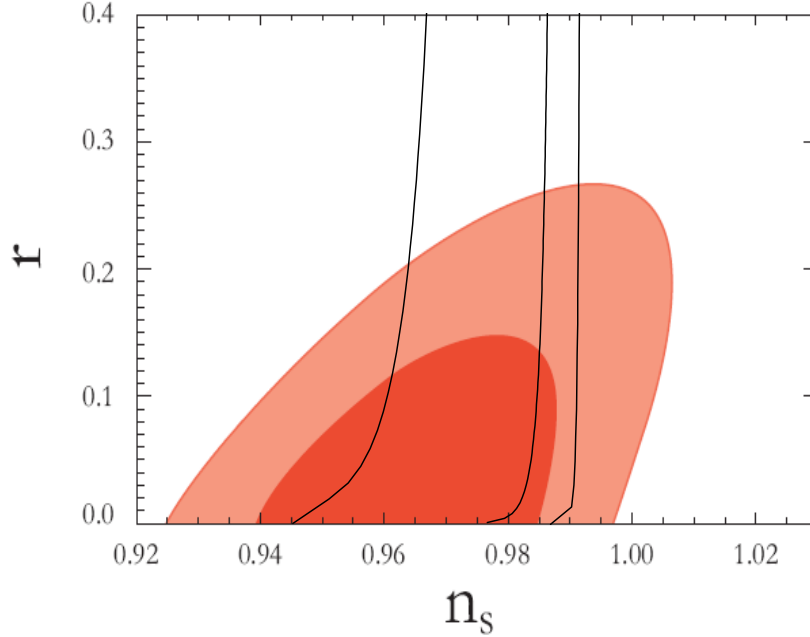


FIG. 2: Evolution of the tensor-scalar ratio r versus the scalar spectrum index n_s in the strong dissipative regime, for three different values of the parameter C_ϕ for the GB regime, where $q = 2/3$. Here, we have used $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$ and $\beta_{2/3}^2 = 10^{-5} m_p^{-2/3}$.

In particular the Fig.(2) shows (for the strong dissipative regime) the dependence of the tensor-scalar ratio on the spectral index for the GB regime, i.e. $q = 2/3$. Here, we have used different values for the parameter C_ϕ . From left to right $C_\phi = 10^6$, $C_\phi = 10^7$ and $C_\phi = 10^8$, respectively. In order to write down values that relate n_s and r , we used Eqs. (46) and (48). Also we have used the values $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$, and $\beta_{2/3}^2 = 10^{-3} m_p^{-2/3}$, respectively. From Eqs.(44) and (48), we observed numerically that for $C_\phi = 10^7$, the curve $r = r(n_s)$ for WMAP-7 years enters the 95% confidence region for $r \simeq 0.26$, which corresponds to

the number of e-folds, $N \simeq 539$. The curve $r = r(n_s)$ enters the 68% confidence region for $r \simeq 0.13$ corresponds to $N \simeq 467$, in this way the GB regime is viable also for large values of N , in the strong dissipative regime. We also noted that the parameter C_ϕ , which is bounded from bellow, $C_\phi > 10^6$, the model is well supported by the data.

V. CONCLUSIONS

In this paper we have studied the warm-intermediate inflationary universe model in braneworld cosmologies, in the weak and strong dissipative regimes. We have also obtained explicit expressions for the corresponding, dissipation coefficient Γ , scalar potential $V(\phi)$, the number of e-folds N , power spectrum of the curvature perturbations $P_{\mathcal{R}}$, q-tensor-scalar ratio r_q and scalar spectrum index n_s .

In order to bring some explicit results we have taken the constraint $r_q - n_s$ plane to first-order in the slow roll approximation. When $\Gamma < 3H$ warm inflation occurs in the so-called weak dissipative regime. In this case, the dissipation coefficient $\Gamma \propto \phi^{-\alpha_1}$ for intermediate inflation and the scalar potential $V(\phi) \propto \phi^{-\alpha_2}$. In particular, we noted that for the GB regime ($q = 2/3$) the parameter C_ϕ , which is bounded from bellow, $C_\phi > 10^7$, the model is well supported by the data as could be seen from Fig.(1). Here, we have used the WMAP seven year data, and we have taken the values $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$, and $\beta_{2/3}^2 = 10^{-5}m_p^{-2/3}$, respectively. On the other hand, when $\Gamma > 3H$ warm inflation occurs in the so-called strong dissipative regime. In this regime, the dissipation coefficient Γ present a dependence proportional to $[\log(\phi)]^{-\mu_2}/\phi^2$ and the scalar potential $V(\phi) \propto [\ln(\phi)]^{-2(1-f)/(q\mu_1)}$. In particular, the Fig.(2) shows that for the values of the parameter $C_\phi = 10^6, 10^7$ or 10^8 , the model is well supported by the WMAP 7-data, when the values $q = 2/3$, $f = 3/5$, $\kappa = 1$, $C_\gamma = 70$, and $\beta_{2/3}^2 = 10^{-3}m_p^{-2/3}$, are taken.

In this paper, we have not addressed the non-Gaussian effects during warm inflation (see e.g., Refs.[39, 47]). A possible calculation from the non-linearity parameter f_{NL} , would give new constrains on the parameters of the model. On the other hand, when $R < 1$ the dissipation does not affect the dynamics of inflation. When $R > 1$ the dissipation does control the dynamics of the inflaton field. Given that the ratio R will also evolve during inflation, we may have also models where we start say with $R < 1$ but end in $R > 1$, or the other way round. In this paper, we have not treated these dynamics. We hope to return to

these points in the near future.

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